

Dynamic Interactions between Visual Working Memory and Saccade Target Selection

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Appendix: Neurodynamic Model

Dynamic Neural Fields and Interactions

The model constitutes an integrated dynamical system composed of five DNFs and three discrete dynamic nodes. In the equations below, we identify each field by a unique index: v for visual sensory field, sa for spatial attention field, sm for saccade motor field, fa for feature attention field, fm for feature memory field, fix for fixation node, gc for gaze change node, and r for saccade reset node. Parameters of projections between fields are identified by two indices, the first signifying the target, the second the source of the projection.

For numeric simulations, the fields are sampled at discrete, equidistant points and the activation values are updated in fixed time steps of 2 ms using the Euler method. The time constant for all field equations is $\tau = 20$ ms. The surface feature dimension is sampled with 174 units, with separate regions for color hue values (144 units) and gray values (30 units). The feature space in each of these regions is defined in a circular manner, without any local interactions between the regions. The spatial dimension is sampled with 301 units, covering a range from approximately -15° to 15° in retinocentric space (with logarithmic mapping of stimulus positions onto this spatial dimension, as detailed below).

The output of all fields is computed from the field activation u via the logistic function

$$f(u) = \frac{1}{1 + \exp(-\beta u)} \quad (1)$$

with steepness parameter β . To compute interactions within and between fields, this output is convolved with an interaction kernel k , which for all

one-dimensional fields takes the general form of a difference of Gaussians with global inhibition:

$$k(x) = \frac{w^{\text{exc}}}{\sqrt{2\pi}\sigma^{\text{exc}}} \exp\left(-\frac{x^2}{2(\sigma^{\text{exc}})^2}\right) - \frac{w^{\text{inh}}}{\sqrt{2\pi}\sigma^{\text{inh}}} \exp\left(-\frac{x^2}{2(\sigma^{\text{inh}})^2}\right) - w^{\text{gi}} \quad (2)$$

Parameters w^{exc} , w^{inh} , and w^{gi} specify interaction strengths, σ^{exc} and σ^{inh} specify kernel widths (always given in field units as specified above). Noise in the field activation is drawn independently for each Euler step and for each location from a normal distribution, then smoothed with a normalized Gaussian kernel k_{noise} with width $\sigma_{\text{noise}} = 2$ along both the spatial and the feature dimension:

$$\xi(\vec{x}, t) = [k_{\text{noise}} * \nu(\cdot, t)](\vec{x}) \quad \text{with} \quad \nu(\vec{x}, t) \sim \mathcal{N}(0, 1) \quad (3)$$

The parameter values of the fields and their lateral interactions are given in Table ??, the parameters of interactions between fields are given in Table ??.

Model architecture

The two-dimensional visual sensory field is governed by the field equation

$$\begin{aligned} \tau \dot{u}_v = & -u_v(x, y) + h_v + i_v(x, y) + [k_{v,v} * f(u_v)](x, y) \\ & + [k_{v,\text{sa}} * f(u_{\text{sa}})](x) + [k_{v,\text{fa}} * f(u_{\text{fa}})](y) + q_v \xi(x, y). \end{aligned} \quad (4)$$

Note that the dependence of activation on time is omitted in all field equations for brevity. The visual sensory field receives driving visual input i_v , detailed below. Activation is modulated by lateral interactions described by the two-dimensional kernel $k_{v,v}$, featuring local surround inhibition along the spatial dimension and global inhibition along the surface feature dimension:

$$\begin{aligned} k_{v,v}(x, y) = & \frac{w_{v,v}^{\text{exc}}}{2\pi\sigma_{v,v}^{\text{exc,spt}}\sigma_{v,v}^{\text{exc,fttr}}} \exp\left(-\frac{x^2}{2(\sigma_{v,v}^{\text{exc,spt}})^2} - \frac{y^2}{2(\sigma_{v,v}^{\text{exc,fttr}})^2}\right) \\ & - \frac{w_{v,v}^{\text{inh}}}{\sqrt{2\pi}\sigma_{v,v}^{\text{inh,spt}}} \exp\left(-\frac{x^2}{2(\sigma_{v,v}^{\text{inh,spt}})^2}\right). \end{aligned} \quad (5)$$

Activation is further modulated by feedback from the spatial and feature attention fields, smoothed by the respective interaction kernels.

The field equation for the feature attention field is

$$\begin{aligned} \tau \dot{u}_{\text{fa}}(y) = & -u_{\text{fa}}(y) + h_{\text{fa}} + i_{\text{fa}} + [k_{\text{fa,fa}} * f(u_{\text{fa}})](y) \\ & + [k_{\text{fa,v}} * O_v^{\text{fttr}}](y) + [k_{\text{fa,fm}} * f(u_{\text{fm}})](y) + q_{\text{fa}} \xi(y) \end{aligned} \quad (6)$$

with the surface feature input from the visual sensory field computed by integrating the field output over the spatial dimension, $O_v^{\text{ftr}}(y) = \int f(u_v(x, y))dx$. The feature attention field provides input to the feature WM field with field equation

$$\begin{aligned} \tau \dot{u}_{\text{fm}}(y) = & -u_{\text{fm}} + h_{\text{fm}} + i_{\text{fm}} + [k_{\text{fm},\text{fm}} * f(u_{\text{fm}})](y) \\ & + [k_{\text{fm},\text{fa}} * f(u_{\text{fa}})](y) + q_{\text{fm}}\xi(y). \end{aligned} \quad (7)$$

Here, i_{fm} is a global excitatory control input that determines when new activation peaks can form in the field.

In the spatial pathway, the spatial attention field is governed by the field equation

$$\begin{aligned} \tau \dot{u}_{\text{sa}}(x) = & -u_{\text{sa}}(x) + h_{\text{sa}} + i_{\text{sa}}(x) + p_{\text{sa}}(x) + [k_{\text{sa},\text{sa}} * f(u_{\text{sa}})](x) \\ & + [k_{\text{sa},\text{v}} * O_v^{\text{spt}}](x) + [k_{\text{sa},\text{sm}} * f(u_{\text{sm}})](x) \\ & + W_{\text{sa},\text{fix}}(x)f(u_{\text{fix}}) - W_{\text{sa},\text{gc}}(x)f(u_{\text{gc}}) - w_{\text{sa},\text{r}}^{\text{gi}}f(u_{\text{r}}) + q_{\text{sa}}\xi(x). \end{aligned} \quad (8)$$

The field receives direct visual input i_{sa} (purely spatial) and is modulated during the saccade and memory test task by constant preshape p_{sa} reflecting task instructions and prior knowledge (described in detail below). It also receives spatial input from the visual sensory field, computed by integrating over the surface feature dimension, $O_v^{\text{spt}}(x) = \int f(u_v(x, y))dy$. Input from the fixation node and gaze change node modulate activation in the foveal region of the field (around zero) through weight patterns

$$W_{\text{sa},\text{fix}}(x) = 2.25 \cdot \exp\left(-\frac{x^2}{2(\sigma_{\text{sa},\text{sa}}^{\text{exc}})^2}\right) \quad (9)$$

and $W_{\text{sa},\text{gc}} = -W_{\text{sa},\text{fix}}$. These two nodes are driven only by external control inputs reflecting task instructions, yielding the simple dynamic equations:

$$\tau \dot{u}_{\text{fix}} = -u_{\text{fix}} + h_{\text{fix}} + i_{\text{fix}} + w_{\text{fix},\text{r}}^{\text{gi}}f(u_{\text{r}}) + q_{\text{fix}}\xi \quad (10)$$

$$\tau \dot{u}_{\text{gc}} = -u_{\text{gc}} + h_{\text{fix}} + i_{\text{gc}} + w_{\text{fix},\text{r}}^{\text{gi}}f(u_{\text{r}}) + q_{\text{gc}}\xi \quad (11)$$

Both the spatial attention field and the nodes are suppressed by inhibitory input from the saccade reset node during a gaze change.

The field equation for the saccade motor field is

$$\begin{aligned} \tau \dot{u}_{\text{sm}}(x) = & -u_{\text{sm}}(x) + h_{\text{sm}} + [k_{\text{sm},\text{sm}} * f(u_{\text{sm}})](x) \\ & + [k_{\text{sm},\text{sa}} * o_{\text{sa}}^{\text{fov}}](x) - w_{\text{sm},\text{r}}^{\text{gi}}f(u_{\text{r}}) + q_{\text{sm}}\xi(x). \end{aligned} \quad (12)$$

In the input from the spatial attention field, the foveal region is suppressed (so no saccade signal will be created for already fixated stimuli), yielding

$$o_{\text{sa}}^{\text{fov}}(x) = \left(1 - \exp\left(-\frac{x^2}{2(\sigma_{\text{sm,sa}}^{\text{exc}})^2}\right) \right) f(u_{\text{sa}}(x)). \quad (13)$$

The saccade reset node globally suppresses activation in the saccade field once it becomes active. Its dynamics are described by the equation

$$\tau \dot{u}_{\text{r}} = -u_{\text{r}} + h_{\text{r}} + w_{\text{r,r}}^{\text{exc}} f(u_{\text{r}}) + w_{\text{r,sm}}^{\text{exc}} \int f(u_{\text{sm}}(x)) dx + q_{\text{r}} \xi. \quad (14)$$

Visual Stimuli

For each visual stimulus j with screen position p_j and size l_j , the spatial pattern on the screen is reproduced as a step function

$$h_j(x) = \begin{cases} 1, & \text{if } |x - p_j| \leq \frac{1}{2}l_j \\ 0, & \text{otherwise} \end{cases} \quad (15)$$

This pattern is then transformed onto a retinocentric pattern m_j (with current fixation point x_{fix}) as

$$m_j(x) = h_j(\text{sign}(x)\zeta(\exp(\chi|x|) - 1) - x_{\text{fix}}), \quad (16)$$

with scaling parameters $\zeta = 100 \text{ px}$ and $\chi = \frac{\ln\left(\frac{450 \text{ px}}{\zeta} + 1\right)}{150}$. This spatial pattern is then smoothed with a normalized Gaussian kernel $k_{\text{v,in}}$ with width $\sigma_{\text{v,in}} = 2.5$. It is then expanded to a two-dimensional pattern by multiplying it with a Gaussian pattern over the space of color hue values, centered on the stimulus color c_j and with width $\sigma_c = 4$. The temporal pattern for each stimulus is phasic-tonic, with the phasic component dependent on the stimulus start time $t_{j,\text{start}}$. The complete visual input for the visual sensory field is the sum of all stimulus patterns:

$$i_{\text{v}}(x, y, t) = \sum_j \left(5 \cdot \exp\left(-\frac{t - t_{j,\text{start}}}{100 \text{ ms}}\right) + 10 \right) [k_{\text{v,in}} * m_j](x) \cdot \exp\left(-\frac{(y - c_j)^2}{2\sigma_c^2}\right) \quad (17)$$

The visual input to the spatial attention field, intended to reflect direct visual input from the lateral geniculate nucleus to the superior colliculus, is purely phasic. It is based on the same pattern m_j used above, now smoothed

with difference-of-Gaussians kernel $k_{\text{sa,in}}$ with a global inhibitory component that reduces input strength when multiple stimuli are present:

$$i_{\text{sa}}(x, t) = \sum_j 7.5 \cdot \exp\left(-\frac{t - t_{j,\text{start}}}{100 \text{ ms}}\right) [k_{\text{sa,in}} * m_j](x) \quad (18)$$

While a saccade is in progress, all visual input is set to zero.

Preshape

The preshape for the saccade task pre-activates the spatial attention field in those regions where the target stimulus may appear, and suppresses it at the possible remote distractor locations. To compute the excitatory preshape pattern, we average over the stimulus patterns m_{t_1}, \dots, m_{t_n} for all possible eccentricities of the target stimulus (in steps of one pixel), smoothed with the kernel $k_{\text{sa,in}}$ specified above. For blocks of trials with a remote distractor stimulus, the stimulus pattern m_d for the distractor is subtracted, otherwise this is omitted. The patterns for the two possible directions of target and distractor from the fixation point (left or right) are added up:

$$p_{\text{sacc}}(x) = \sum_{\text{dir}=\{l,r\}} \left(\frac{2.6}{n} \sum_{k=1}^n [k_{\text{sa,in}} * m_{t_k}^{\text{dir}}](x) - 1.2 [k_{\text{sa,in}} * m_d^{\text{dir}}](x) \right) \quad (19)$$

The preshape for the memory simply pre-activates the locations of the two memory test stimuli (left and right), based on their stimulus patterns m_{mt}^l and m_{mt}^r :

$$p_{\text{mt}}(x) = 1.25 \left([k_{\text{sa,in}} * m_{\text{mt}}^l](x) + [k_{\text{sa,in}} * m_{\text{mt}}^r](x) \right) \quad (20)$$

Saccade metrics

A simulated saccade is assumed to start at the time t_{start} at which the output of the saccade reset node first exceeds a threshold $\theta_{\text{start}} = 0.25$, and ends at the time t_{end} at which the nodes output falls below $\theta_{\text{end}} = 0.05$. The saccade amplitude s (in pixels on the screen) is determined by integrating the output of the saccade motor field over the whole time that a supra-threshold activation peak is present in that field (the integration thus begins before the saccade start time t_{start}). The output signal from each field location is scaled in this integration to reflect the stimulus eccentricity it represents,

using the same mapping from field positions (retinocentric with logarithmic scaling) to screen positions as used in computing the visual input:

$$s = 0.0025 \text{ px} \int_{t_{\text{start}}}^{t_{\text{end}}} \int f(u_{\text{sm}}(x, t)) (\text{sign}(x)\zeta(\exp(\chi|x|) - 1)) dx dt \quad (21)$$

Simulation time course and control inputs

The simulation time course closely emulates the psychophysical experiment, with task instructions reflected by external control inputs. These control inputs modulate the behavior of the model through simple changes in field activation levels and thereby increase the model’s behavioral flexibility. We assume that these inputs would be provided by cortical areas involved in cognitive control (such as prefrontal cortex), which are outside the scope of the present model.

At the beginning of each trial, the activation of all fields and nodes in the model are reset to their resting levels. The memory sample stimulus is then activated for 300 ms (in simulation time), and the global control input $i_{\text{fm}} = 2.5$ is applied to the feature WM field during this time. This input allows the feature WM field to form self-sustained peaks and reflects the task instruction to memorize the color presented in this phase of the trial. After this period, the fixation stimulus is activated. In preparation of the saccade task, the preshape p_{sacc} is applied to the spatial attention field and the gaze change node receives an input $i_{\text{gc}} = 5$ to facilitate saccade initiation. The preshape reflects the expectation of the target stimulus position (from task instructions and practice trials), and the input to the gaze change node reflects the instructions to rapidly make a saccade to the upcoming target stimulus. The saccade target and distractor stimuli (if applicable) are then activated with a delay of 700 ms, and turned off again 200 ms after the first saccade has been initiated.

In preparation for memory test task, the gaze direction is then reset manually to the fixation stimulus 300 ms after the first saccade (this is done to avoid any biases in the memory test induced by initial gaze direction). At this time, the preshape pattern in the spatial attention field is switched from p_{sacc} to p_{mt} . After another 100 ms, global activation in the feature attention field and feature WM field are increased by $i_{\text{fa}} = 2$ and $i_{\text{fm}} = 1.5$, respectively. This brings the system into a visual search mode, where feature match dominates saccade target selection. After another 100 ms, the two memory test stimuli are activated. The trial ends when the system has made a saccade towards either of them.

field index	h	β	q	w^{exc}	σ^{exc}	w^{inh}	σ^{inh}	w^{gi}
v (ftr/spt)	-5	1	0.25	10	5 / 2.5	1	- / 6.25	0
fa	-3.5	4	0.25	10	4	18	8	0.1
fm	-5	4	0.5	30	3	37.5	9	0.1
sa	-2	1	0.25	15	12	0	-	0.3
sm	-5	4	0.5	42	8	0	-	0.95
fix	-5	1	0.2	0	-	0	-	0
gc	-5	1	0.2	0	-	0	-	0
r	-5	4	0.2	3	-	0	-	0

Table 1: Field parameters and parameters of lateral interactions.

projection index	w^{exc}	σ^{exc}	w^{inh}	σ^{inh}	w^{gi}
fa, v	0.4	4	0	-	0
v, fa	3.75	6	0	-	0
fm, fa	2.5	6	0	-	0
fa, fm	8.5	8	0	-	0
sa, v	1.5	10	1	25	0
v, sa	2.5	12	0	-	0
sm, sa	7.25	10	0	-	0
sa, sm	7.25	10	0	-	0.1
r, sm	0.4	-	0	-	0
sa, r	0	-	0	-	12
sm, r	0	-	0	-	12
fix, r	0	-	0	-	5
sa, in	1.25	10	0.5	25	0.015

Table 2: Parameters of interactions between fields.